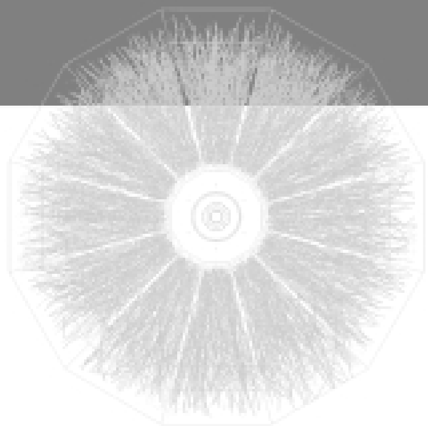

More

Millikan's Oil-Drop Experiment

Millikan's measurement of the charge on the electron is one of the few truly crucial experiments in physics and, at the same time, one whose simple directness serves as a standard against which to compare others. Figure 3-4 shows a sketch of Millikan's apparatus. With no electric field, the downward force on an oil drop is mg and the upward force is bv. The equation of motion is

$$mg - bv = m\frac{dv}{dt} \tag{3-10}$$

where b is given by Stokes' law:

$$b = 6\pi\eta a \tag{3-11}$$

and where η is the coefficient of viscosity of the fluid (air) and a is the radius of the drop. The terminal velocity of the falling drop v_f is

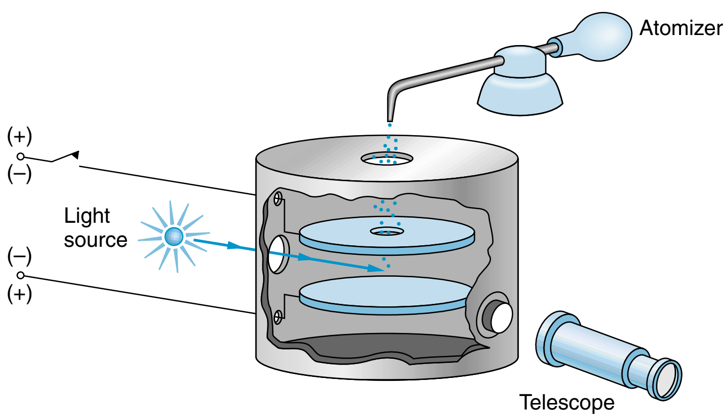

Fig. 3-4 Schematic diagram of the Millikan oil-drop apparatus. The drops are sprayed from the atomizer and pick up a static charge, a few falling through the hole in the top plate. Their fall due to gravity and their rise due to the electric field between the capacitor plates can be observed with the telescope. From measurements of the rise and fall times, the electric charge on a drop can be calculated. The charge on a drop could be changed by exposure to x rays from a source (not shown) mounted opposite the light source.

(*Continued*)

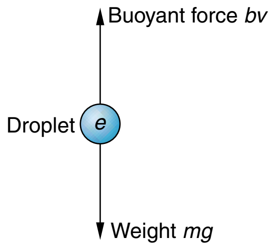

Fig. 3-5 An oil droplet carrying an ion of charge e falling at terminal speed, i.e., $mg = bv$.

$$v_f = \frac{mg}{b} \tag{3-12}$$

(see Figure 3-5). When an electric field $\mathscr{E}$ is applied, the upward motion of a charge q_n is given by

$$q_n\mathscr{E} - mg - bv = m\frac{dv}{dt}$$

Thus the terminal velocity v_r of the drop rising in the presence of the electric field is

$$v_r = \frac{q_n\mathscr{E} - mg}{b} \tag{3-13}$$

In this experiment, the terminal speeds were reached almost immediately, and the drops drifted a distance L upward or downward at a constant speed. Solving Equations 3-12 and 3-13 for q_n, we have

$$q_n = \frac{mg}{\mathscr{E}v_f}(v_f + v_r) = \frac{mgT_f}{\mathscr{E}}\left(\frac{1}{T_f} + \frac{1}{T_r}\right) \tag{3-14}$$

where $T_f = L/v_f$ is the fall time and $T_r = L/v_r$ is the rise time.

If any additional charge is picked up, the terminal velocity becomes v'_r, which is related to the new charge q'_n by Equation 3-13:

$$v'_r = \frac{q'_n\mathscr{E} - mg}{b}$$

The amount of charge gained is thus

$$\begin{aligned} q'_n - q_n &= \frac{mg}{\mathscr{E}v_f}(v'_r - v_r) \\ &= \frac{mgT_f}{\mathscr{E}}\left(\frac{1}{T'_r} - \frac{1}{T_r}\right) \end{aligned} \tag{3-15}$$

The velocities v_f, v_t, and v'_r are determined by measuring the time taken to fall or rise the distance L between the capacitor plates.

If we write $q_n = ne$ and $q'_n - q_n = n'e$ where n' is the *change* in n, Equations 3-14 and 3-15 can be written

$$\frac{1}{n}\left(\frac{1}{T_f} + \frac{1}{T_r}\right) = \frac{\mathscr{E}e}{mgT_f} \tag{3-16}$$

and

$$\frac{1}{n'}\left(\frac{1}{T'_r} - \frac{1}{T_r}\right) = \frac{\mathscr{E}e}{mgT_f} \tag{3-17}$$

(*Continued*)

To obtain the value of e from the measured fall and rise times, one needs to know the mass of the drop (or its radius, since the density is known). The radius is obtained from Stokes' law using Equation 3-12.

Notice that the right sides of Equations 3-16 and 3-17 are equal to the same constant, albeit an unknown one, since it contains the unknown e. The technique, then, was to obtain a drop in the field of view and measure its fall time T_f (electric field off) and its rise time T_r (electric field on) for the unknown number of charges n on the drop. Then, for the *same* drop (hence, same mass m), n was changed by some unknown amount n' by exposing the drop to the x-ray source, thereby yielding a new value for n; and T_f and T_r were measured. The number of charges on the drop was changed again and the fall and rise times recorded. This process was repeated over and over for as long as the drop could be held in view (or until the experimenter became tired), often for several hours at a time. The value of e was then determined by finding (basically by trial and error) the *integer* values of n and n' that made the left sides of Equations 3-16 and 3-17 equal to the same constant for all measurements using a given drop.

Millikan did experiments like these with thousands of drops, some of nonconducting oil, some of semiconductors like glycerine, and some of conductors like mercury. In no case was a charge found that was a fractional part of e. This process, which you will have the opportunity to work with in solving the problem below using actual data from Millikan's sixth drop, yielded a value of e of 1.591×10^{-19} C. This value was accepted for about 20 years, until it was discovered that x-ray diffraction measurements of N_A gave values of e that differed from Millikan's by about 0.4 percent. The discrepancy was traced to the value of the coefficient of viscosity η used by Millikan, which was too low. Improved measurements of η gave a value about 0.5 percent higher, thus changing the value of e resulting from the oil-drop experiment to 1.601×10^{-19} C, in good agreement with the x-ray diffraction data. The modern "best" values of e and other physical constants are published periodically by the International Council of Scientific Unions. The currently accepted value of the electron charge is[8]

$$e = 1.60217733 \times 10^{-19}\text{C} \qquad \textbf{3-18}$$

with an uncertainty of 0.30 parts per million. Our needs in this book are rarely as precise as this, so we will typically use $e = 1.602 \times 10^{-19}$ C. Note that, while we have been able to measure the value of the quantized electric charge, there is no hint in any of the above as to *why* it has this value, nor do we know the answer to that question now.

Hardly a matter of only historical interest, Millikan's technique is currently being used in an ongoing search for elementary particles with fractional electric charge by M. Perl and co-workers.

PROBLEM

The accompanying table shows a portion of the data collected by Millikan for drop number 6 in the oil-drop experiment. (a) Find the terminal fall velocity v_f from the table using the mean fall time and the fall distance (10.21 mm). (b) Use the density of oil $\rho = 0.943$ g/cm^3 = 943 kg/ m^3, the viscosity of air $\eta = 1.824 \times 10^{-5}$ N·s/m^2, and $g = 9.81$ m/s^2 to calculate the radius a of the oil drop from Stokes' law as expressed in Equation 3-12. (c) The correct "trial value" of n is filled in in column 7. Determine the remaining correct values for n and n', in columns 4 and 7, respectively. (d) Compute e from the data in the table.

(*Continued*)

Rise and fall times of a single oil drop with calculated number of elementary charges on drop							
1 T_f	2 T_r	3 $1/T'_r - 1/T_r$	4 n'	5 $(1/n')(1/T'_r - 1/T_r)$	6 $1/T_f + 1/T_r$	7 n	8 $(1/n)(1/T_r - 1/T_f)$
11.848	80.708				0.09655	18	0.005366
11.890	22.366	0.03234	6	0.005390	0.12887	24	0.005371
11.908	22.390						
11.904	22.368						
11.882	140.566	0.03751	7	0.005358	0.09138	17	0.005375
11.906	79.600	0.005348	1	0.005348	0.09673	18	0.005374
11.838	34.748	0.01616	3	0.005387	0.11289	21	0.005376
11.816	34.762						