2006-10-06 Problem Solutions

A very useful number for doing these problems is \( hc = 1240 \text{ eV \cdot nm} \).

**Problem 1: Tipler 3-30**

Using Equation 3-36,

\[
(1) \quad 0.95 = \frac{\hbar}{e} \left( \frac{c}{435.8 \times 10^{-9} m} \right) - \frac{\Phi}{e}
\]

\[
(2) \quad 0.38 = \frac{\hbar}{e} \left( \frac{c}{546.1 \times 10^{-9} m} \right) - \frac{\Phi}{e}
\]

Subtracting (2) from (1),

\[
0.57 = \frac{\hbar c}{e 10^{-9}} \left( \frac{1}{435.8} - \frac{1}{546.1} \right)
\]

Solving for \( \hbar \) yields: \( \hbar = 6.56 \times 10^{-34} \text{ J\cdot s} \). Substituting \( \hbar \) into either (1) or (2) and solving for \( \Phi/e \) yields: \( \Phi/e = 1.87 \text{ eV} \). Threshold frequency is given by \( h/\Phi = \Phi/e \) or

\[
f = \left( \frac{\Phi}{e} \right) \left( \frac{e}{\hbar} \right) = \frac{(1.87 \text{ eV}) (1.60 \times 10^{-19} \text{ C})}{6.56 \times 10^{-34} \text{ J\cdot s}} = 4.57 \times 10^{14} \text{ Hz}
\]

**Problem 2: Tipler 3-36**

The energy of this photon is \( E = 0.511 \text{ MeV} \), so its momentum is \( p = 0.511 \frac{\text{MeV}}{c} \).

The wavelength is \( p = \frac{\hbar}{\lambda} \), so \( \lambda = \frac{\hbar}{p} \), and its initial wavelength is

\[
\lambda_i = \frac{\hbar}{0.511 \text{ MeV} \times 0.511 \text{ MeV} / \text{c}} = \frac{1240 \text{ eV \cdot nm}}{0.511 \text{ MeV} / \text{c}} = 2.427 \times 10^{-3} \text{ nm} = 2.427 \text{ pm}.
\]

The final wavelength is given by the Compton effect formula \( \lambda_f - \lambda_i = \frac{h}{m_c c} (1 - \cos \theta) \).

For an electron the Compton wavelength is (p. 147) \( \frac{\hbar}{m_c c} = 0.00243 \text{ nm} \). Therefore the final wavelength, for scattering at 110 degrees is

\[
\lambda_f = 2.427 \times 10^{-3} \text{ nm} + (0.00243 \text{ nm}) (1 - \cos 110^\circ) = 5.688 \times 10^{-3} \text{ nm}.
\]

This gives the final momentum of the photon to be
\[
p_f = \frac{h}{\lambda_f} \quad \text{or} \quad cp_f = \frac{hc}{\lambda_f} = \frac{1240 \text{ eV} \cdot \text{nm}}{5.688 \times 10^{-3} \text{ nm}} = 0.218 \text{ MeV}.
\]
Therefore the momentum of the final (scattered) photon is \( p_f = 0.218 \frac{\text{MeV}}{c} \). The energy of the scattered photon is \( E_f = p_f c = 0.218 \text{ MeV} \).

The energy of the initial photon was given as \( E_i = 0.511 \text{ MeV} \), and the initial electron at rest had energy 0.511 MeV; the total energy of the initial state is 1.022 MeV. In the final state the photon had energy 0.218 MeV, so by energy conservation the recoiling electron had the rest of it, which \( E_e = (1.022 - 0.218) \text{ MeV} = 0.804 \text{ MeV} \) (this is total energy).

To find the direction of the recoiling electron, first find the magnitude of its momentum from the energy momentum relation \( E^2 = (mc^2)^2 + (pc)^2 \). Turn it around and apply it to the final state of the electron, \( p_{f,e} = \sqrt{[E_f^2 - (mc^2)^2]^{1/2}} = \sqrt{(0.804)^2 - (0.511)^2} \) MeV = 0.621 MeV. So the momentum of the recoiling electron is \( p_{f,e} = 0.621 \text{ MeV}/c \).

To get the angle we have to look at the picture of the scattered photon and the recoiling electron. The problem says that the scattered photon comes out at 110 degrees, so it looks something like this.

Write the momentum conservation law for the \( \gamma \)-component of momentum.

\[
0 = p_{f,e} \sin \varphi - p_f \sin 110^\circ = 0.621 \sin \varphi - 0.218 \sin 110^\circ.
\]
Solve this for the angle of the recoiling electron

\[
\varphi = \arcsin \left( \frac{0.218 \sin (110^\circ)}{0.621} \right) = \arcsin (0.330) = 19.3^\circ.
\]

**Problem 3: Tipler 3-38**

a) The energy of the incident photons is

\[
E_i = hf_i = \frac{hc}{\lambda_i} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.0711 \text{ nm}} = 1.747 \times 10^4 \text{ eV}.
\]

b) The wavelength of the incident photons is given to be \( \lambda_i = 0.0711 \text{ nm} \). The change in wavelength is given by the Compton scattering equation

\[
\Delta \lambda = \frac{h}{mc} (1 - \cos \theta) = (0.00243 \text{ nm}) (1 - \cos 180^\circ) = 0.00486 \text{ nm},
\]
and so the final wavelength is \( \lambda_f = \lambda_i + \Delta \lambda = (0.0711 + 0.00486) \text{ nm} = 0.07596 \text{ nm} \).

c) The energy is \( E_f = \frac{hc}{\lambda_f} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.07596 \text{ nm}} = 1.632 \times 10^4 \text{ eV} \).
d) By energy conservation, the difference of energy between the initial and final photons is the electron energy. \( E_e = E_i - E_f = 1.128 \times 10^3 \text{ eV} \).

**Problem 4: Tipler 3-55**

(a) The nonrelativistic expression for the kinetic energy of the recoiling nucleus is

\[
E_k = \frac{p^2}{2m} = \frac{(15 \text{ MeV/c})^2}{2 \times 12u} \left( \frac{1u}{931.5 \text{ MeV/c}^2} \right) = 1.10 \times 10^4 \text{ eV}
\]

Internal energy \( U = 15 \text{ MeV} - 0.0101 \text{ MeV} = 14.9899 \text{ MeV} \)

(b) The nucleus must recoil with momentum equal to that of the emitted photon, about 14.98 MeV/c.

\[
E_k = \frac{p^2}{2m} = \frac{(14.98 \text{ MeV/c})^2}{2 \times 12u} \left( \frac{1u}{931.5 \text{ MeV/c}^2} \right) = 1.00 \times 10^{-2} \text{ eV}
\]

\[
E_q = U - E_k = 14.9899 \text{ MeV} - 0.0100 \text{ MeV} = 14.9799 \text{ MeV}
\]