(a) \[ K.E. = E - V(x) = \frac{h^2}{2mL^2} - \frac{h^2}{2mc^4} x^2 \]

\[ = \frac{h^2}{2mc^2} \left( 1 - \frac{x^2}{L^2} \right) \]

(b) Classical turning point is where \( E = V(x) \), i.e. there is no kinetic energy. Thus, \( K.E = 0 \)

\[ \Rightarrow \frac{1 - \frac{x^2}{L^2}}{L^2} = 0 \Rightarrow x = \pm L. \]

(c) \( V(x) = \frac{1}{2} m \omega^2 x^2 \) (Harmonic Oscillator Potential Energy)

\[ \Rightarrow \frac{h^2}{2mc^4} x^2 = \frac{m \omega^2}{2} \quad \therefore \quad \omega^2 = \frac{h^2}{mL^4} \Rightarrow \omega = \frac{h}{mL}. \]

Thus \( E = \frac{h^2}{2mc^2} = \left( \frac{h}{mc^2} \right)^2 = \frac{h}{2} \omega. \)
6.6 For a free electron $V(x) = 0$ so the TISE gives

$$\frac{-\hbar^2}{2m} \frac{d^2 \Psi}{dx^2} = E \Psi$$

Now $\Psi(x) = A \sin \left(2.5 \times 10^{10} x\right)$

$$\frac{d^2 \Psi(x)}{dx^2} = (2.5 \times 10^{10})^2 \Psi(x)$$

Using this in the TISE gives

$$\frac{(2.5 \times 10^{10})^2 \hbar^2}{2m} \Psi = E \Psi$$

For a free particle $E = \text{Kinetic Energy} = \frac{p^2}{2m}$ so

$$p^2 = 2mE = 2m \left(2.5 \times 10^{10}\right)^2 \frac{\hbar^2}{2m} = \left(6.25 \times 10^{20} \hbar \right)^2$$

$$p = 2.5 \times 10^{10} \hbar = 2.5 \times 10^{10} \times 6.63 \times 10^{-34} \frac{\text{kg m/s}}{2 \pi}$$

$$= 2.64 \times 10^{-24} \text{ kg m/s}$$

(b) $E = \frac{p^2}{2m} = \left(2.64 \times 10^{-24} \text{ kg m/s}\right)^2 = 3.82 \times 10^{-18}$

$$= \frac{3.82 \times 10^{-18}}{1.60 \times 10^{-19} \text{ J/F}} = 23.9 \text{ eV}$$
(c) \[ \lambda = \frac{h}{P} = \frac{6.63 \times 10^{-34} J s}{2.64 \times 10^{-24} W s/m} = 2.5 \times 10^{-10} m = 0.25 \mu m \]
Normalization is just a statement that uses probability:

\[
\int_{-\infty}^{\infty} e^{-x^2} \, dx = 1
\]

\[
\therefore \int_{-a}^{a} e^{-ax^2} \, e^{-(hx-wt)} \, dx = 1
\]

\[
= A^2 \int_{-a}^{a} \, dx = A^2 2a = 1
\]

\[
\Rightarrow A = \frac{1}{\sqrt{2a}}
\]

Obviously if \(a \to \infty\) then \(A \to 0\) so normalization is not valid here. Such a case can be handled using limit procedures that are labelled box normalization.
NO. For example, we know from Example 6.1 on p. 250 that for a free particle, the wavefunction

\[ \Psi(x) = A \sinh x + B \cosh x \]

is a solution to the TISE for any value of \( A + B \).

Thus take \( A = i \quad B = 1 \quad \Rightarrow \)

\[ \Psi(x) = \cosh x + i \sinh x = e^{ikx} \]

which is not real but is a solution to the TISE.